NOLTR 63-133

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THE PREDICTION OF VERY-LOW EED FIRING PROBABILITIES

NOL

4 SEPTEMBER 1963

UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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THE PREDICTION OF VERY-LOW EED FIRING PROBABILITIES

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ABSTRACT: EED functioning probability levels needed for estimating weapon safety and reliability cannot be measured exactly. Existing interpretations of "All-Fire" and "No-Fire" levels often are seriously in error or even invalid. Practical procedures for reducing errors in estimating extreme functioning probability levels are presented: proper sampling, proper instrumentation, optimization of data collection procedures, and selection of proper statistical tools.

PUBLISHED JANUARY 1964

EXPLOSION DYNAMICS DIVISION
EXPLOSIONS RESEARCH DEPARTMENT
U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

NOLTR 63-133 4 September 1963

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The present report, with minor variations was presented at the Second HERO Congress, 30 April - 2 May 1963. Because the subject material is of broad interest it warrants a wider distribution than it might get by appearing only in the Proceedings of the Congress. As the title indicates the report deals with low functioning probability levels and therefore is of interest in weapon safety considerations. However, by symmetry and analogy the concepts presented herein are equally useful for high functioning probability level estimates needed for weapon reliability considerations.

The work leading to the writing of this report was supported by Task NOL 443.

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C. J. ARONSON
By direction

CONTENTS

ILLUSTRATIONS

INTRODUCTION

1. The assessment of the hazards of electromagnetic radiation to explosive ordnance centers largely about the determination of the probability of inadvertantly firing electro-explosive devices by RF energy. A major part of this paper will be devoted to expounding the thesis that, in general, the firing probability of EED's has not been determined with sufficient accuracy to allow the hazard assessment to be made with realistic precision. A second objective will be to set forth techniques by which the firing probabilities of EED's can be determined with greater accuracy at input stimuli associated with low firing response. Finally, it is hoped that this paper will give enough insight into the problem of predicting EED response, that the presently-used ambiguous concepts of "All-Fire" and "No-Fire" points will be eradicated.

2. The safety-design goal for Naval weapon fuzes has long been that there be no more than one weapon in a million wherein the warhead charge shall be initiated unintentionally at any time from manufacture to target delivery. This fact is cited to indicate the high level of safety desired in the Navy's detonating munitions and as point of reference for establishing safety goals for today's modern weapons of larger range and potency. The demonstration of such probability levels is virtually impossible by direct testing methods. Inferential methods must be used to amass relevant data and to make acceptable estimates.

RESPONSE OF EED'S

3. The major portion of the HERO problem arises from the RF vulnerable EED's used throughout weapon systems. It is necessary that EED sensitivity to electrical energy (power) be related to the ambient electrical environment if an estimate of safety is to be made. Furthermore the sensitivity (probability of response to a particular intensity of environment) is needed at a very low probability of firing level.

4. In order to understand the various methods of predicting extreme firing probability levels a passing understanding of probability distribution functions is of benefit. It has been found that the values of many naturally occurring phenomena can be sufficiently well described for many purposes by the normal (Gaussian) distribution function, for example, the heights of 1162 Vassar students in 1958 (Figure 1). Here we can see that the Gaussian curve describes the distribution of heights quite well except at the upper tail where only one or two students would be expected to be over 6 feet tall. Actually there were five.

5. The normal distribution can be represented graphically in a number of different ways: the bell-shaped frequency curve (Figure 2), the cumulative curve or ogive curve (Figure 3), and the cumulative curve transformed to a straight line by plotting in an appropriate probability space (Figure 4). While these three forms appear to be different they are equivalent. They all demonstrate the fact that the function is asymptotic at both extremes. That is, the probability of initiation does not become zero until the stimulus is decreasing without limit. Similarly 100% probability is approached but never actually attained. Whether or not this asymptotic property is an accurate description of the nature of EED's will be brought out in later discussions, particularly when considering "All-Fire" and "No-Fire" concepts.

6. The cumulative function is the more useful one for making estimates of extreme firing probability levels, particularly when in the straight-line form*. From the straight line relation it is obvious that a particular distribution can be fully identified by two data points: either a particular functioning-stimulus-and-probability point and the slope of the line, or else two functioning-stimulus-and-probability points.

THE METHODS FOR ESTIMATING EXTREME FUNCTIONING PROBABILITY POINTS

7. Extreme functioning probability points for EED's have usually been estimated by extrapolating on the basis of an experimentally determined mean and standard deviation. As is the way with extrapolations, this process can lead to seriously faulty answers. There are certain assumptions inherent in the extrapolation:

a. That the sample size is adequate

b. That the sample is used efficiently. (This is controlled by the design of the experiment and the accuracy of the apparatus.)

c. That the sample is representative of the batch or lot from which it was taken.

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* It should be noted that the point of intersection of the line with the 50% firing level is designated as μ, the fifty per cent firing stimulus, while σ, the standard deviation is the reciprocal of the slope at μ. It is a statistical convention to reserve the Greek symbols μ and σ for expressing the properties of the parent population and m and s respectively as the estimates based on measurements on samples from the population.

d. That the sampled batch (or lot) is representative of <u>all</u> possible batches (or lots) of that particular EED.

e. That the distribution function employed in making the extrapolation does in fact describe the sensitivity of the EED.

8. If extrapolation is so dangerous why is it used? Why are not the extreme functioning responses measured directly rather than by this tenuous method? The obvious reason is that the number of EED's (often very costly) needed to make a direct measurement is prohibitive. Suppose that one is interested in knowing the current which will cause not more than one in ten thousand EED's to fire. It would be necessary to observe thirty thousand trials without a single fire before one could say with reasonable assurance (a risk of one chance in twenty) that the current will not cause more than one in ten thousand EED's to fire.

CHOICE OF DISTRIBUTION FUNCTION

9. There is no other way out than extrapolation in the present state of the art. There is considerable hope that knowledge of the electro-thermal parameters coupled with the hot-spot theory[1]* will eventually make it possible to establish safe currents through EED bridgewires. Until such time we must therefore select a distribution function which will be used as a basis for making the desired estimates.

10. To our knowledge only in two instances have there been sufficiently detailed tests made to give a quantitative picture of the sensitivity distribution function of EED's. One of these was carried out at Franklin Institute on 4362 carbon bridge EED's[2]. The other, carried out on 7890 wire bridge EED's, was reported at the last HERO Congress by two of the present authors[3]. This work leads to the following conclusions concerning the proper choice of distribution function:

a. The horizontal axis should be in logarithmic units, i.e., log current, log energy, or log voltage.

b. The Gaussian probability curve is not a good fit since it predicts too high a reliability above the 50% point and too low a probability (greater safety than actually exists) below the 50% point.

c. The logistic distribution function[4] does not give an accurate fit but at least it seems to err on the side of over conservatism.

* References will be found on page 15

It has been the authors' recommendation that in the absence of more definitive information, the log-logistic distribution function be used for making extreme functioning probability estimates of wire bridge EED's. Many, if not most, of the estimates currently available in the literature, manufacturing data, and specifications are based on the Gaussian distribution rather than the logistic.

11. In passing, we would like to point out an extra advantage in using the logistic distribution function. Being of the form:

$$L = \ln\frac{p}{q} = \frac{x-\bar{x}}{r}$$

where p is probability, q=1-p, x is the stimulus, $\bar{x}$ is the mean stimulus, and r is the reciprocal of the slope in the logistic probability space, this function can be evaluated with a set of log tables by desk computations and can be programmed very simply for high-speed computers. The cumulative Gaussian function cannot be evaluated in terms of elementary functions and is therefore difficult to incorporate into high-speed computer programs.

LIMITATIONS OF THE BRUCETON DATA-COLLECTION PLAN

12. The Bruceton plan is the most widely used method today for obtaining and analyzing firing data, undoubtedly because of the conservative sample size and the ease of making statistical calculations. But the Bruceton plan is extremely poor for making the large extrapolations needed in the estimation of extreme firing levels. The testing is conducted close to the 50% firing level requiring that the distribution function be very well known to allow long extrapolations. Further, the Bruceton plan tends to give poor estimates of the standard deviation, which is one of the parameters by which the extrapolation is made. Work carried out in England by J. W. Martin of the Royal Armament Research and Development Establishment, Ft. Halstead[5], shows that poor estimates of the standard deviation occur even with reasonable sample sizes (100 firings, for example). The situation is depicted graphically in Figures 5 and 6, which were obtained by making high-speed computer Bruceton runs with a known normal distribution. The bell-shaped curves show the expected relative frequency distributions of estimates of the standard deviation. The histograms show the distributions of the estimates that actually were observed. The horizontal coordinates of each bell-curve/histogram pair are expressed as a ratio of the estimate of the standard deviation to its true value. Inherent in the Bruceton process is the fact that the precisions of the estimates of the mean and standard deviation are affected by the location of the mean with respect to the test

height when the step size is <u>large</u> compared to the standard deviation. From the curves it can be seen that the standard deviation is more seriously underestimated when the step size is <u>small</u> compared to the standard deviation and to a lesser degree affected by the location of the mean with respect to the test height. The probability of underestimating the mean is not negligible with large samples (100 units) and is much greater with small sample sizes (25 units). An estimate of the standard deviation equal to 50% of its true value (which could happen quite often in the 25 unit case) would lead to an estimate of a hazard of 1/1,000,000 when the true risk was 1/115 or an estimate of 1/1,000 when the true risk was 1/16. Underestimating the standard deviation, of course, will give overly optimistic predictions of both safety and reliability. To underestimate the true standard deviation will give unreasonable estimates of the various firing points as shown below where extrapolation is made from the fifty per cent firing level.

Predicted Firing Probability $s = 0.5\sigma$	True Firing Probability Per Cent
0.0001	0.873
0.001	1.644
0.01	3.148
0.10	6.118
1.0	12.24
10.0	26.08
20.0	33.68

13. Other Monte Carlo studies carried out independently at the Naval Ordnance Laboratory show dramatically the imprecision to be expected with small sample size Bruceton determinations. Fifteen Bruceton type tests, twenty shots per test, were carried out using a population with a known mean of 5.60 and a known standard deviation of 2.00. Figure 7 compares the individual estimates of the 90, 50 and 10 per cent points with the known population. Note, for instance, that the predicted 10 per cent point of the eighth run is higher than the predicted 90 per cent point of the fifteenth run.

14. Still another instance can be quoted. Recently one thousand Primers Mk 114 were fired under constant current conditions at the Naval Ordnance Laboratory. A preliminary 30 shot Bruceton run yielded a 50 per cent point of 1.8 milliamperes which is virtually the same as was observed for the 1000 shot rundown tests. The Bruceton predicted 1 per cent firing level was 62.7 milliamperes. Experimentally, in the rundown at a level of 62.5 milliamperes, 3.85% functioning was actually observed.

15. The original paper on the analysis of the Bruceton method gives a test for normality. This is based on what is known as the Chi-square test. Unfortunately use of this test will never prove that the distribution is normal. Failure to pass the test is taken as proving that the distribution is not normal whereas passing the test merely shows that it could be, but is not necessarily, normal. What is even more disquieting, unless the distribution differs greatly from the normal, a large number (i.e., many hundreds) of trials will be required to establish the non-normality of the distribution. The usual Bruceton test does not have a sufficient number of trials to do this.

16. Another very serious limitation lies in the fact that the Bruceton test collects the data near the fifty per cent point. There are many possible distribution functions which would match the data collected over this range in a satisfactory manner but which would differ to a very considerable extent for low or high per cent response points. Figure 8 shows several such curves fitted to data of a typical Bruceton test. To state this problem in another way, if the true distribution function should be even slightly peaked, or slightly flattened, near the mean (as compared with the assumed function) then we find that the true and assumed function would have diverged greatly by the time the extreme functioning probability levels were reached For instance, in Figure 9 we see a case where the true function had longer tails than the assumed function. Even if we had a perfect Bruceton, one in which the mean and the standard deviation measured at the mean were without error, we would be making overly optimistic estimates at both ends. In Figure 10 we show a way we can collect the data to reduce the effect of lack of agreement of the true and assumed functions when making estimates at low probability levels. It should be noted that the assumed distributions fitted to the data in Figure 10 are depicted as straight line while the true (but unknown) distribution is shown as a curvedline.

17. For emphasis. we repeat that the Bruceton collection plan should not be used for this work. We realize that the Bruceton plan is an old friend, a popular and tried-and-trusty tool. It has its uses. But it is not the tool for this job. Here we must find some way of allocating our samples so that we approach more closely to the functioning level we wish to estimate but yet not get so far away from the mean that we get almost meaningless (saturated*) data.

*By saturated data we mean a level at which all fails or else all fire were observed. Since we compute p=no. of fires/no. of trials we would find p=o or ∞ respectively. We would be unable to plot it in our straight line cumulative probability space.

BATCH-TO-BATCH AND LOT-TO-LOT VARIABILITY

18. It is usual to treat electro-explosive devices as if each had individual fixed characteristics. We think in terms of a fixed firing energy for a particular design of EED under given conditions, and publish data to show what its characteristics are. For example, the Mk 1 Squib requires 10,000 ergs for 50% firing under adiabatic input. But it is known that when each of two different manufacturers make the squib the firing characteristics will probably be different. These differences can occur from slightly different bridge wire lengths, bridge wire diameters, fineness of explosive about the bridge wire, etc. It has even been found that when the same manufacturer makes the same EED over a period of time the firing characteristics may vary widely. An example of this is shown for the bridge-explosive combination of the Mk 114 Primer, manufactured by the same company with an interval of approximately 10 years between the two manufactured batches, (Figure 11).

19. That an EED cannot be considered to be of fixed firing characteristics is thus apparent. This fact should not be surprising when it is realized that manufacturers using the same plant, equipment, and processes can turn out both acceptable and unacceptable lots of EED's during a given production. What should not be expected or assumed is that a given variety of EED is invariant in its characteristics; consequently safety estimates cannot be realistically made assuming invariant firing properties.

SAMPLING ERROR

20. In estimating the firing characteristics of a group of EED's, test firing must be made with a sample taken from the group. The results of this test firing can be taken as estimates for the group as a whole. These estimates will, of course, differ from the true, but unknown, values. These differences constitute the sampling errors. Their possible magnitudes for random samples can be estimated by standard statistical methods. These estimates would be in the form of a statement that the error is probably less than a certain amount. In a certain proportion of the tests, five per cent for example, the error will be greater than this amount.

21. Remember that these error estimates will be valid only if the sample was taken randomly. A random sample is one in which the selection of the individual item does not depend upon any property of the item. Every item must have an equal chance of being selected as a member of the sample. In many cases the selection of a truly random sample becomes difficult if not actually impossible. For example, we cannot obtain a random sample of all Mk 114 Primers. The sample must, for practical

reasons be drawn from the items on hand at a particular time and place. The result is that one does not estimate the characteristics of the Mk 114 Primer but of some particular lot or group of these Primers. As was pointed out in the preceding paragraph any attempt to consider these results as a characterization of the Mk 114 Primer is fallacious. This is the same error as that made by the European who thinks that he knows what Americans are like because he has seen a few American movies or has spent a day or two in New York City.

THE "ALL-FIRE" "NO-FIRE" TRAP

22. Over the past few years there has been an increasing tendency to use as concepts for characterizing EED's:

a. The "No-Fire" level, i.e., the largest input stimulus which can be applied to an EED without initiating it.

b. The "All-Fire" level, i.e, the smallest input stimulus which must be applied to an EED to initiate it with certainty.

These concepts are very handy from the engineering standpoint. They may even be true. But to date we have not been able to devise a method for locating these points or levels, or of verifying their existence.

23. The usual process for estimating the "All-Fire" and "No-Fire" levels involves a measurement of response at a number of levels and then an extrapolation in the appropriate direction with an assumed distribution function. But the asymptotic limits of the distribution function are automatically assumed since they are part of the distribution function. And these limits are the desired "No-Fire" and "All-Fire" levels. Since they have been assumed a priori, they cannot be estimated or measured by the assumed distribution function. These limits are $-\infty$ and $+\infty$ for a linear normal or linear logistic function. Since a negative firing energy or power is meaningless the logarithmic transform is usually used. With the logarithmic transformation, the "No-Fire" level becomes zero energy or power and the "All-Fire" level remains $+\infty$. Other distribution functions could easily be conjured up which would be asymptotic at finite non-zero levels, but they would have to be based on absolute knowledge before their limits could be identified with certainty as the desired "All-Fire" and "No-Fire" levels.

24. There is a way of getting an idea whether or not a particular test level is above or below the "All-Fire" level. For instance, let us assume that 3 million units of a particular type of EED

are to be tested at 3.0 amperes. If there is a mixed response (both fires and fails) we know with certainty that the "All-Fire' level, if it exists, is greater than 3.0 amperes. If all of the units respond we may have exceeded the "All-Fire" level. But there is still one chance in twenty that we are at as low a probability as a 0.999,999*. There is an even greater chance that we have not exceeded the "All-Fire" level. But to get this assurance of only one in a million failure level at 95% confidence we had to use a fantastic sample size and we still are not sure that we have reached an "All-Fire" level or even that it exists.

25. The term "No-Fire" is an idea which seems to us to be similar to the idea of an upper boundary to the earth's atmosphere. All of the initiators of a given kind, the entire population in statistical language, correspond in this analogy to the atmosphere. The population of initiators is made up of individuals each of which will have a minimum stimulus requirement just as the atmosphere is made of individual molecules each having a certain height** above the earth. If there is atmosphere at any given height above the earth's surface then there will be some gas molecules at a slightly greater height. A similar situation exists with initiator sensitivity. If a few initiators will fire at a given stimulus, it is impossible to say that a decrease of an erg in the energy or a milliampere in the current would never result in a fire.

26. It is our contention that the terms "All-Fire" and "No-Fire" do not mean literally what they say. Is there, then, a meaning for these terms which, though different from the literal translation, is generally accepted? It is certainly possible that an arbitrary definition could be made and generally accepted in the same way as other empirical parameters such as flash point of oil, yield point of steel, etc. At present there are many interpretations of the concepts ranging from carefully computed points to engineering hunches and guesses.

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*The sample size needed to make this same sort of a prediction at other probabilities can be computed very simply by:

$$N = \frac{3P}{1-P}$$

where N is the number of items in the sample, P is the lower 95% confidence limit for the estimate of the probability of having observed N fires out of N trials.

** For the purposes of this analogy it is immaterial that the minimum stimulus requirement of the initiator is fixed while gas molecule heights vary with time. The atmosphere analogy is restricted to a single instant in time.

27. For instance, we find in the specifications for the Squib Mk 1 that either forty-five or seventy-five items (depending upon the lot size) are to be tested with a current of 200 milliamperes. If none fire the lot is accepted. This 200 milliampere level is quoted as the "No-Fire" current for this initiator. But the ghastly truth is that the chances are five out of eight that forty-five failures would be observed at a level which in reality was the 1% functioning level. Or, to look at it another way, a manufacturer could be producing Mk 1 Squibs which will function 0.1% of the time at this 200 milliampere "All-Fail" and have 19 out of 20 lots accepted.

28. Recently certain groups at the Bureau of Naval Weapons have decided to define the "m-4s" and the "m+4s" levels as the "No-Fire" and "All-Fire" levels respectively. These correspond to probabilities of 32 in a million that an adverse result is expected in either case, i.e., 0.000032 or 0.999968. The choice of 4 standard deviations away from the mean is purely arbitrary. It is a seat-of-the-pants compromise between what one might like to be able to say (one in a million) and what one might be really justified in saying (one in a hundred).

THE WAY OUT

29. By now the picture we have painted must indeed be bleak. We have said that even if a good random sample is taken from the lot being tested there is no assurance that the other lots will not differ from it. We have said that the normal distribution and the Bruceton data-collection plan should not be used to make extrapolations. We have said that the "All-Fire" and "No-Fire" levels probably do not exist or else if they do exist they cannot be determined by direct experiment. We will say further that it will almost always be the case that safety estimates must be made by extrapolating hopefully with a surmised distribution function on data derived from an inadequate sample size taken from a lot which probably will be different from the lot that is used in the weapon.

30. The first purpose of this report is to post a warning as to the nature and degree of the problem. The second is to show ways to reduce the likelihood of making faulty estimates:

a. Get the largest possible sample size

b Make sure that the sampling was random

c. Make sure that the firing instrumentation is in good order. (The inherent experimental error should be known.)

d. Carry out the firing according to a plan which has been optimized for the purpose of making the desired estimate.

e. Choose the distribution function which will yield conservative estimates

f. Estimates should be made realistically in view of the limitations inherent in the process. Assumptions should be recognized and stated with the deduced sensitivity figures.

31. What is meant by "largest possible sample size"? How big is a large sample size? As was mentioned previously 1000 Primers Mk 114 were fired under constant-current conditions. The following firing data were obtained:

Current Milliamperes	Observed Response Fires	Fails	Per Cent
84.0	2	0	100.00
77.0	9	1	90.00
70.0	4	9	30.77
67.5	10	35	22.22
64.0	8	171	4.47
62.5	15	375	3.85
61.0	1	349	0.29

Using the log-logistic distribution function, and fitting the data using the least-squares method, the sensitivities are estimated:

Probability of Functioning (per cent)	Predicted Level milliamperes
50.0	71.19
5.0	63.70
1.0	59.85
0.1	54.85
0.01	50.28
0.001	46.09

The 95% tolerance interval (upper and lower confidence band) about the 0.1% point is from 50 to 57 milliamperes. The estimate of the 0.1% point is not particularly precise. Had the sample size been in the order of 100 units this interval would have been over three times larger. Thus it can be seen that a sample of a few hundred units really cannot be expected to give a reliable and precise estimate much beyond the 1% point. It begins to be evident that the definition of a large sample size is: a quantity much larger than one can hope for.

32. The next three items listed in paragraph 30 are quite clear. The necessity for random selection of the sample from the population is a basic tenet of proper experimental and statistical procedures. Next, since we must extrapolate on the basis of some measure of the variability of the sample such as s, the standard deviation, we must be assured that the variability introduced by instrumentation errors is small compared to the variability inherent in the EED. Referance 6 is suggested as a detailed analysis of the quantitative problems of firing EED's by capacitor discharge. And finally, the importance of choosing the right distribution function has been discussed in detail elsewhere in this report.

DESIGN OF A DATA COLLECTION PLAN OPTIMIZED FOR MAKING EXTREME FUNCTIONING PROBABILITY ESTIMATES

33. In order to minimize the importance of assumptions regarding the frequency distribution it is desirable to base these estimates on data taken as close as possible to the per cent point to be determined. The simplest such test would be one which calls for testing at two stimulus levels near to the region in question. One of the two levels will be further from the mean and closer to the desired point than the other. This will be designated the remote stimulus level. The data obtained can then be extrapolated to determine the stimulus associated with the desired per cent point. In planning such an experiment the following conditions should be met:

a. The difference between the stimuli used should not be small compared to the extrapolation distance (the difference between the desired point and the observed remote stimulus)

b. The number of trials at the remote stimulus level and the expected response at this level must be such that the probability of observing either all-fires or all-fails is small

c. The number of trials made at the remote functioning level should be greater than the number of trials at the level closer to the mean. A good choice is to take the number so that the product $np(1-p)$ is the same for both levels where n is the number of trials and p is the expected probability of fire.

34. The general approach in implementing this method is to run a preliminary short Bruceton which is used to estimate two stimulus levels at which the remainder of the sample will be fired. At the first level (the one closer to the mean) the smaller portion of the remaining sample is fired. If the observed response is not too different from what was expected from the Bruceton test, the remainder of the sample is fired

at the previously selected remote level. If the response at the first level differs markedly from the expected response, the test plan is altered to reallocate the sample remainder.

35. The following is a step-by-step procedure for firing two hundred samples:

a. Fire twenty items in a Bruceton test to obtain preliminary estimates of the mean, m. and the standard deviation, s. A log-transform of the dosage (current, potential, energy) is taken as the stimulus.

b. Compute the magnitude of the two test levels; near stimulus (first level) = m-0.[illegible], remote stimulus (second level) = m-1.3s.

c. Test fifty items at the near stimulus level

(1) if five or fewer fires are observed, redefine the near stimulus level as the remote level by continuing firing at this level until one hundred-thirty items are expended. Fire the remaining fifty at m-0.2s.

(2) if more than five fires are observed (the usual circumstance) fire the remaining one hundred-thirty units at the original remote stimulus level.

36. The foregoing procedure is set up for use when a low per cent point is desired. This method can be used for determining a high per cent point by making the appropriate changes. The test levels should be computed by adding, rather than subtracting, the appropriate multiple of s. The criterion for altering test levels, i.e., "five or fewer fires", would be changed to "five or fewer fails".

37. There is one problem in reliability determination which does not have its counterpart at low per cent points. This is the possibility of the presence of duds which would not respond to any stimulus, no matter how great. Obviously if more than one per cent of the population were duds, a 99% functioning point would be a fiction. It could not exist. Figure 12 shows the expected response of a normally distributed population contaminated by 5% duds. As can be seen from the figure, if an experiment were made covering the range from 15 to 8 % response, the data obtained could be represented quite well by a straight line. Hence any test made entirely in this interval could give erroneous predictions for responses above 90%. Ordinarily the proportion of duds is smaller than 5% and is therefore harder to detect. When the Bruceton test is used the absence of duds is implicitly assumed.

38. The data collection plan described above ordinarily yields some centrally-collected Bruceton data plus data at two levels where mixed responses are observed. Because the Bruceton data are collected at the mean and at levels most remote from the desired estimation level it seems best to discard them and to perform the analysis on the two mixed response levels. This analysis can be carried out very simply by plotting the two points in the proper probability space and running a straight line through them.

39. More sophisticated statistical treatments, experimental designs, etc. can be used and in many cases will lead to better answers. Other papers are being prepared at the Naval Ordnance Laboratory which will describe in much greater detail some of the experimental techniques, statistical theory, and high-speed computer programs for data processing and for carrying out Monte Carlo experiments.

CONCLUSIONS

40. It is not possible to estimate precisely the functioning probability levels of EED's at the extremes needed for good safety and reliability estimates. There are practical procedures for reducing estimation errors: these are proper sampling, proper instrumentation, optimization of data collection procedures, and selection of proper statistical tools. There are certain areas, such as lot-to-lot and batch-to-batch variation, for which we cannot at present make adequate correction to our estimates. The gathering and study of relevant data would probably be quite difficult, yet would give information of great value in this type of work. We feel that even though it is not possible to solve all of the problems accurately it is much better to know the inherent limitations and possible sources of serious error in making safety and reliability estimates than it is to go blithely along in blissful ignorance of life as it really is.

REFERENCES

1. NOLTR 61-20, "The Response of Electro-Explosive Devices to Transient Electrical Pulses", I. Kabik, L. A. Rosenthal, A. Solem, 17 Apr 1961.

2. "Statistical Methods in Initiator Evaluation", The Franklin Institute Laboratories for Research and Development, Interim Report No. I-1804-1, Carl Hammer, May 1, 1955.

3. NAVWEPS Report 7347, "Characterization of Squib Mk 1 Mod 0 Determination of the Statistical Model", L. D. Hampton and J. N. Ayres, 30 Jan 1961.

4. "A Statistically Precise and Relatively Simple Method of Estimating the Bio-Assay with Quantal Response on the Logistic Function", Joseph Berkson, Jour. of Amer. Stat. Assoc., 48, 565-599 (1953).

5. Private Communication, J. W. Martin to I. Kabik, Apr 4, 1962.

6. NAVWEPS Report 7308, "Characterization of Squib Mk 1 Mod 0 Capacitor Discharge Sensitivity with Instrumentation", J. N. Ayres, 10 Jan 1961.

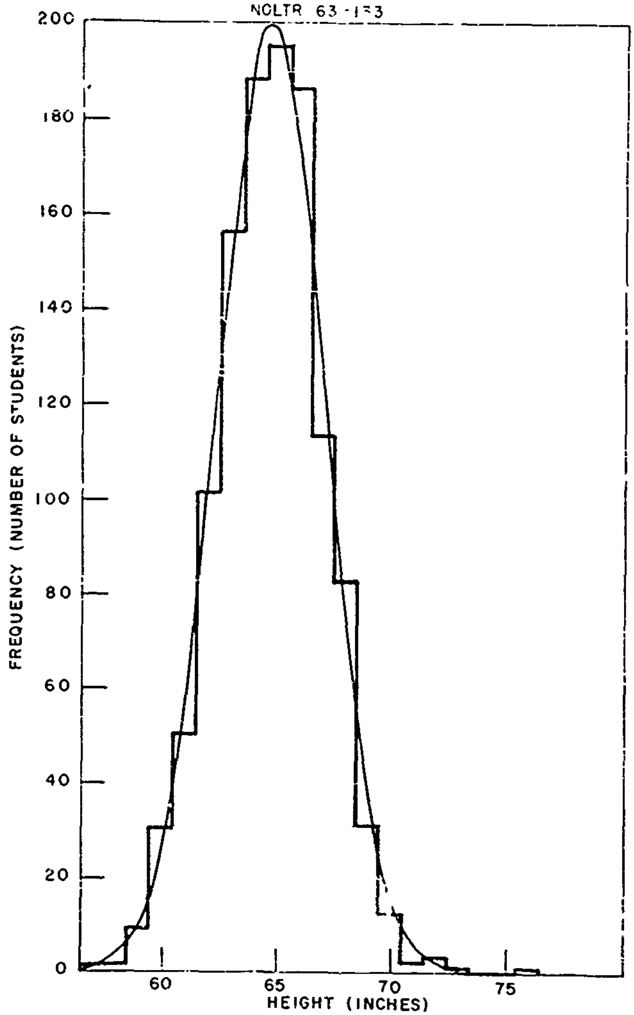

FIGURE I. HEIGHTS OF STUDENTS AT VASSAR COLLEGE, 1958

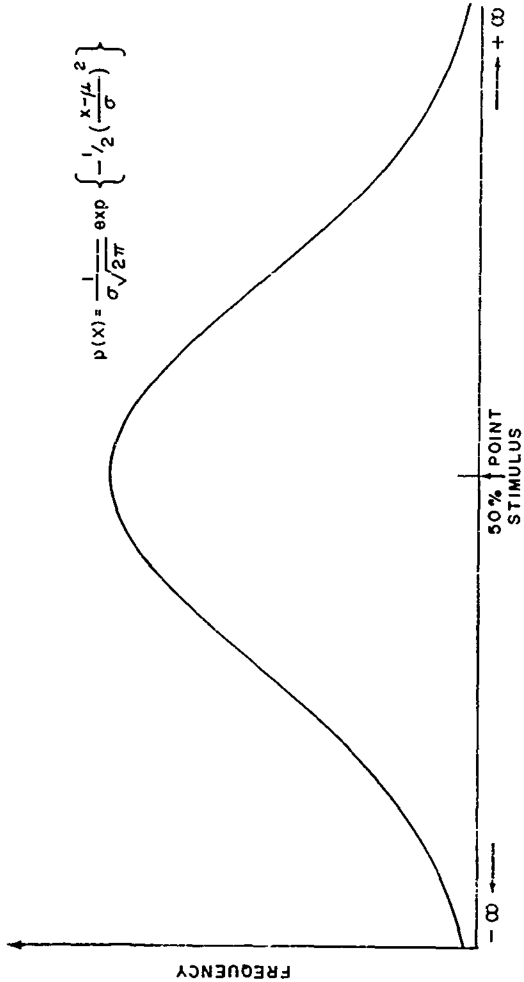

FIGURE 2. DISTRIBUTION FUNCTION DISPLAYED AS A FREQUENCY CURVE (BELL CURVE)

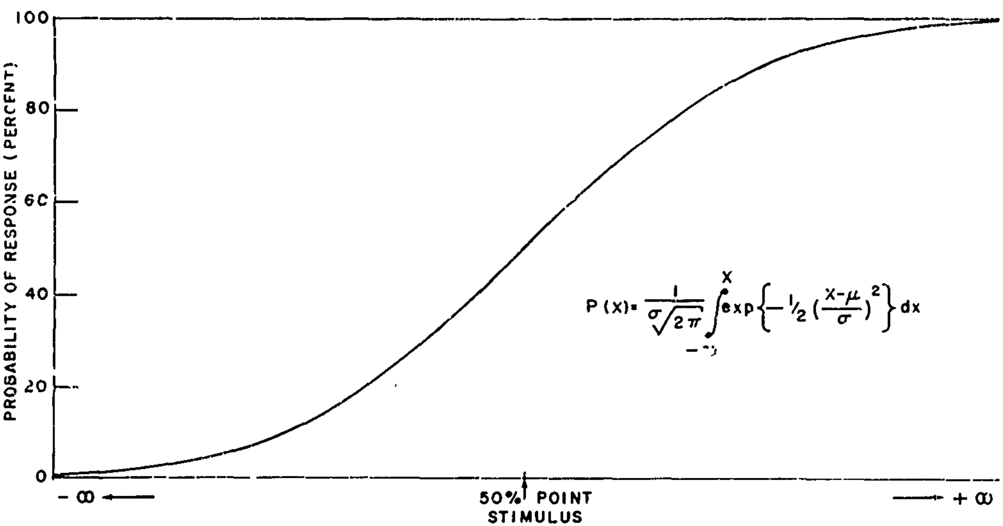

FIGURE 3 CUMULATIVE DISTRIBUTION FUNCTION, DISPLAYED AS A SIGMOIDAL FUNCTION

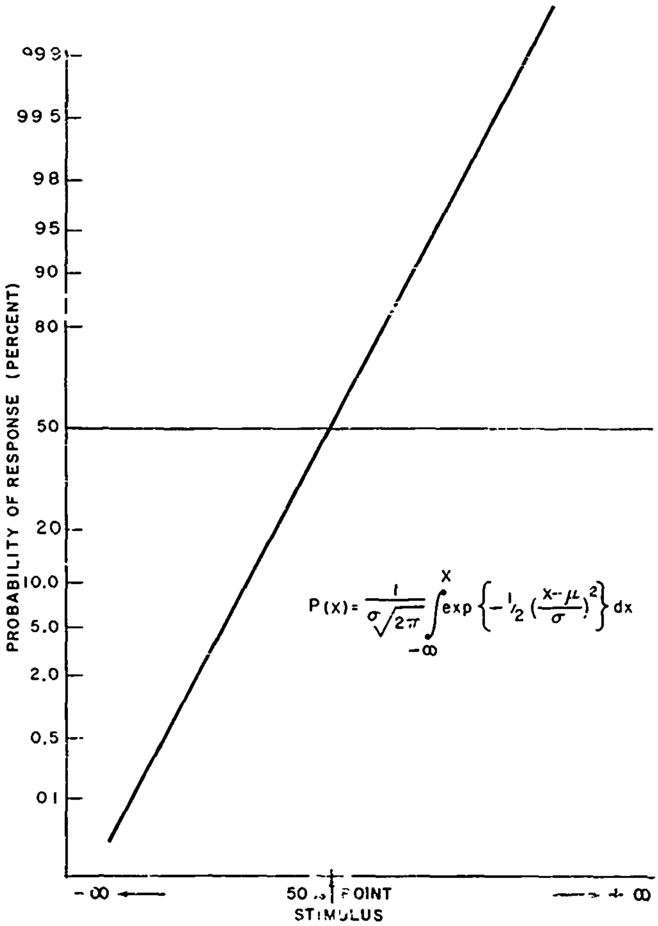

FIGURE 4 CUMULATIVE DISTRIBUTION FUNCTION DISPLAYED AS A STRAIGHT LINE IN A PROBABILITY SPACE

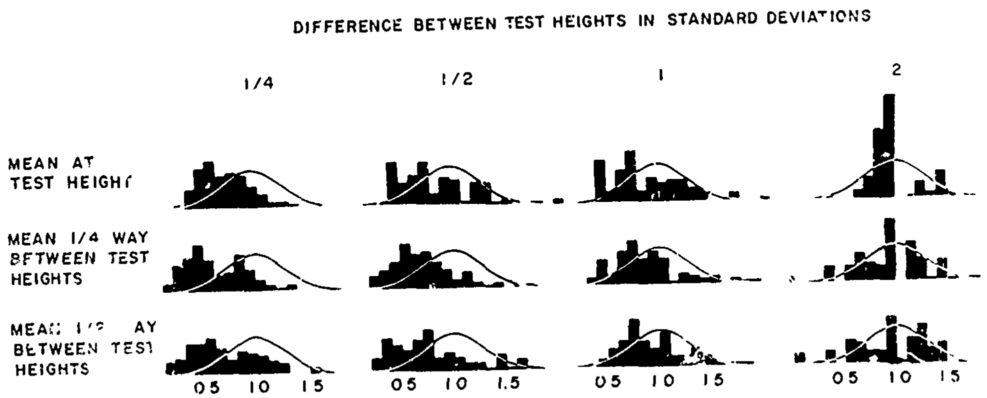

FIGURE 5 SPECTRUM OF STANDARD DEVIATIONS EACH HISTOGRAM GIVES THE RESULT OF 100 TRIALS OF 25 ITEMS. THE X AXIS SHOWS THE RATIO OF "FOUND" TO "TRUE" STANDARD DEVIATION

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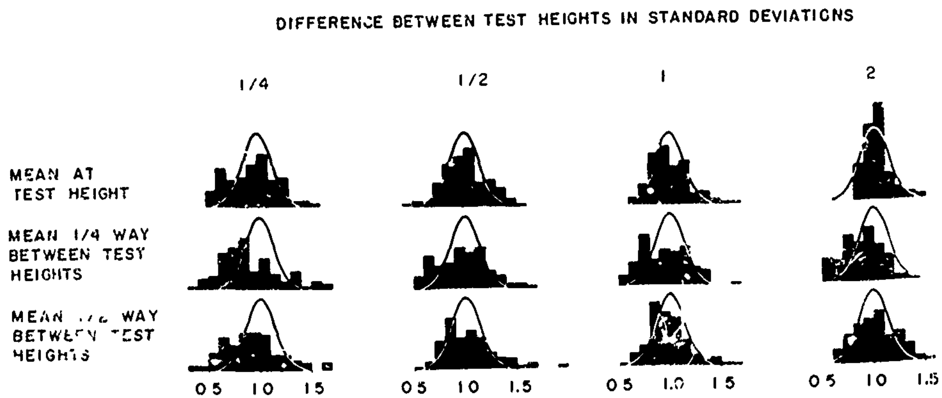

FIGURE 6 SPECTRUM OF STANDARD DEVIATIONS EACH HISTOGRAM GIVES THE RESULT OF 100 TRIALS OF 100 ITEMS THE X AXIS SHOWS THE RATIO OF "FOUND" TO "TRUE" STANDARD DEVIATION

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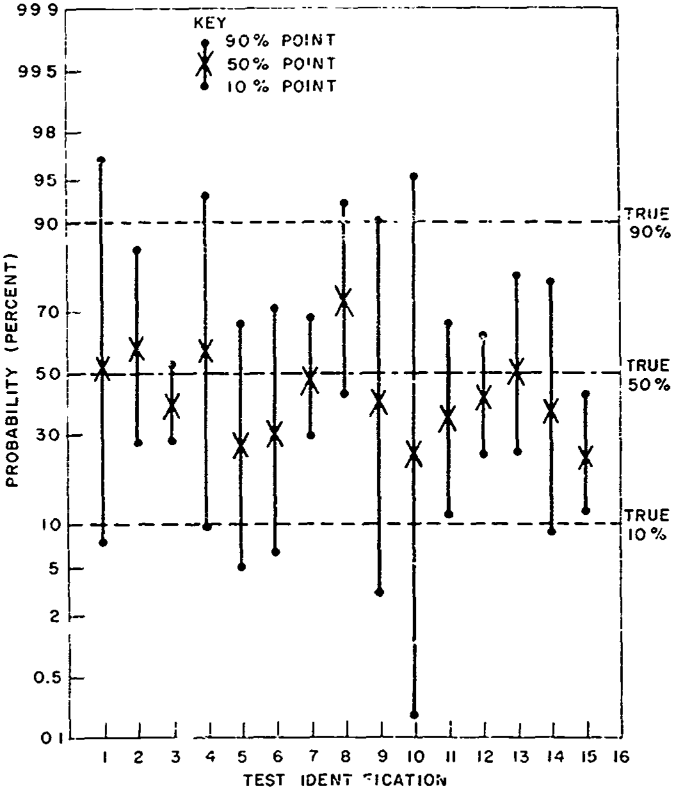

FIGURE 7 VARIABILITY OF ESTIMATES OF 90, 50, AND 10% POINTS BASED ON 20-SHOT BRUCETON SAMPLES FROM A KNOWN POPULATION

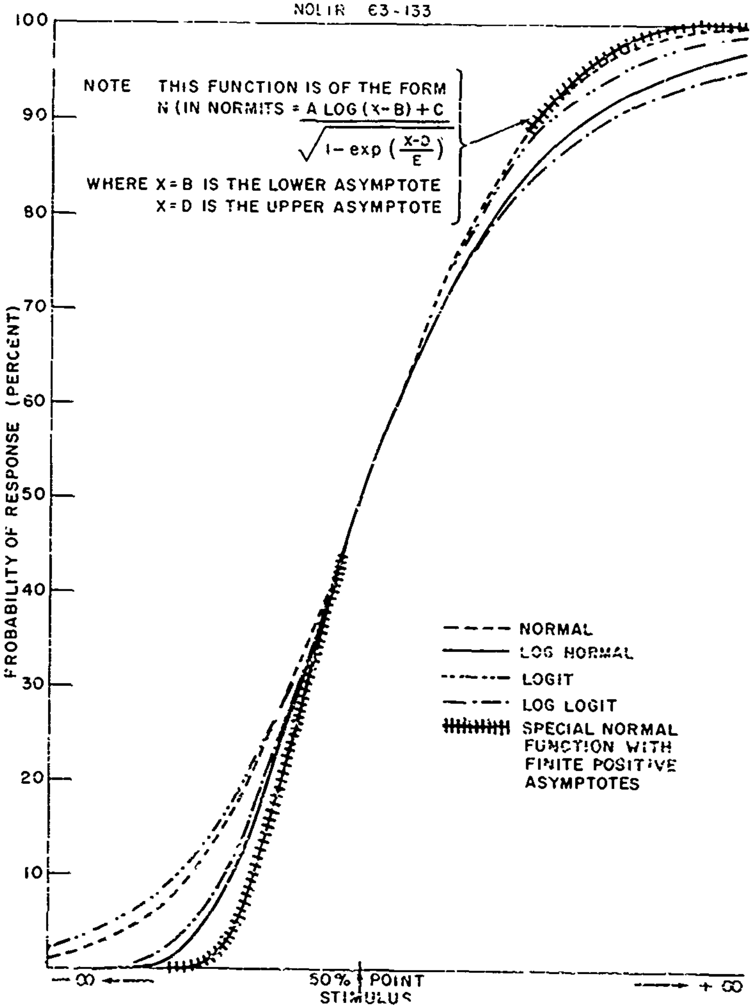

FIGURE 8 COMPARISON OF DISTRIBUTION FUNCTIONS HAVING THE SAME MEAN AND THE SAME SLOPE AT THE MEAN

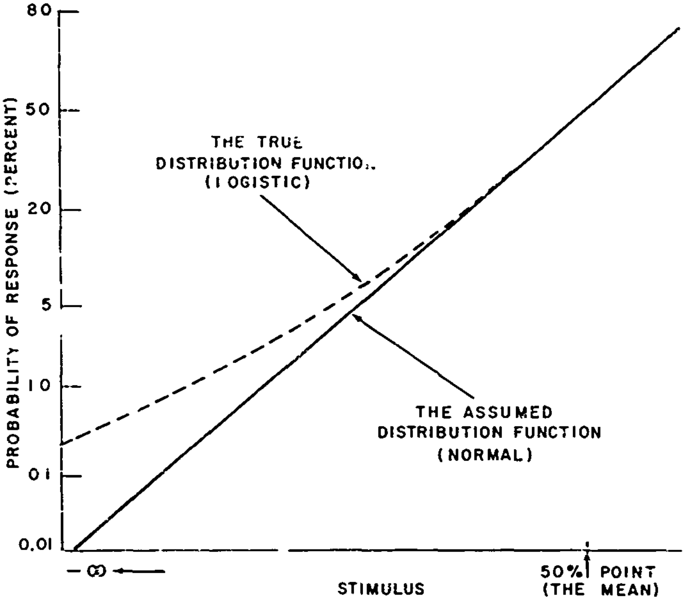

FIGURE 9 THE EFFECT OF FITTING THE ASSUMED DISTRIBUTION FUNCTION TO A MEAN AND SLOPE MEASURED AT THE MEAN

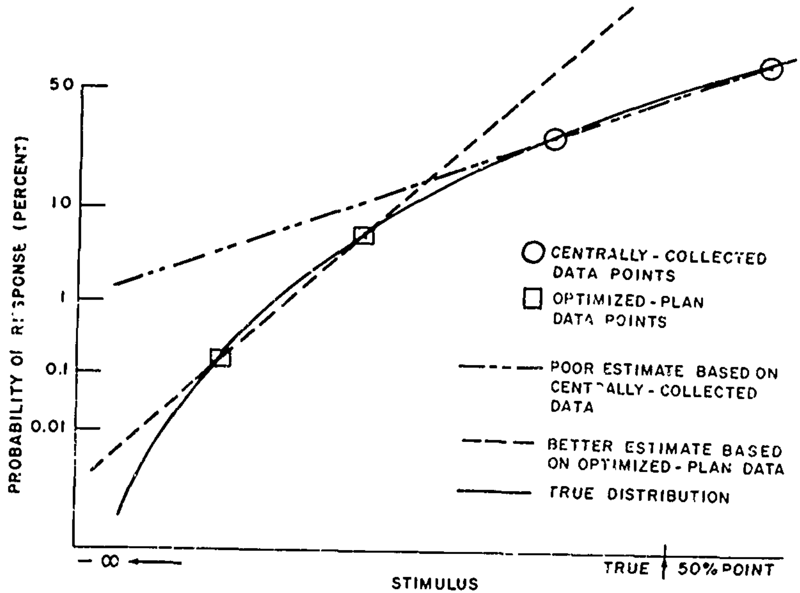

FIGURE 10 OPTIMIZATION OF DATA COLLECTION PLAN TO REDUCE CENTRAL ERROR BY CONCENTRATING DATA BELOW THE 50% POINT FOR SAFETY ESTIMATES

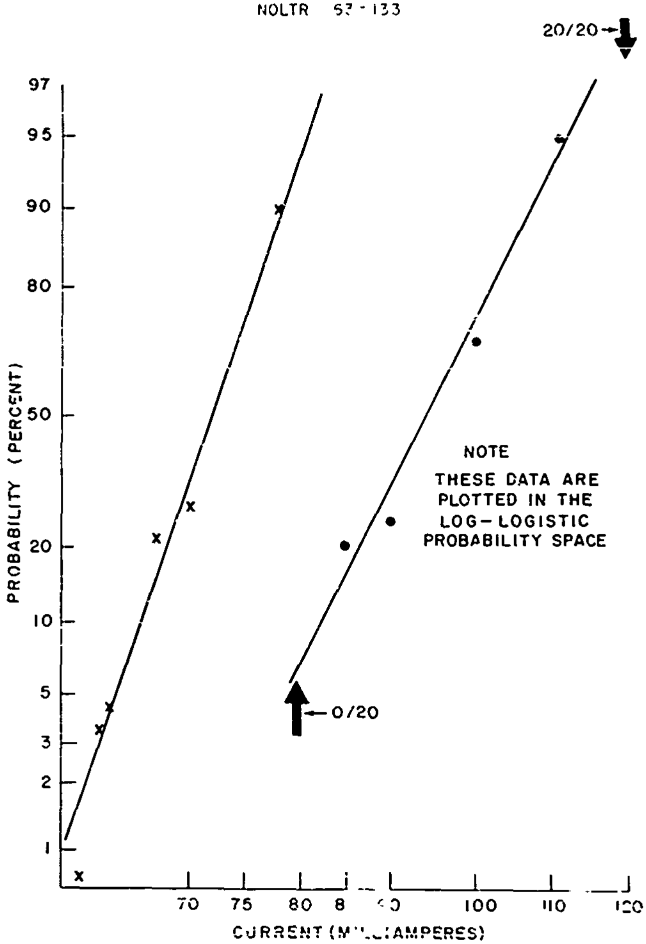

FIGURE 11 LOT-TO-LOT VARIABILITY OF THE CONSTANT-CURRENT SENSITIVITY OF MK 114-TYPE PRIMERS

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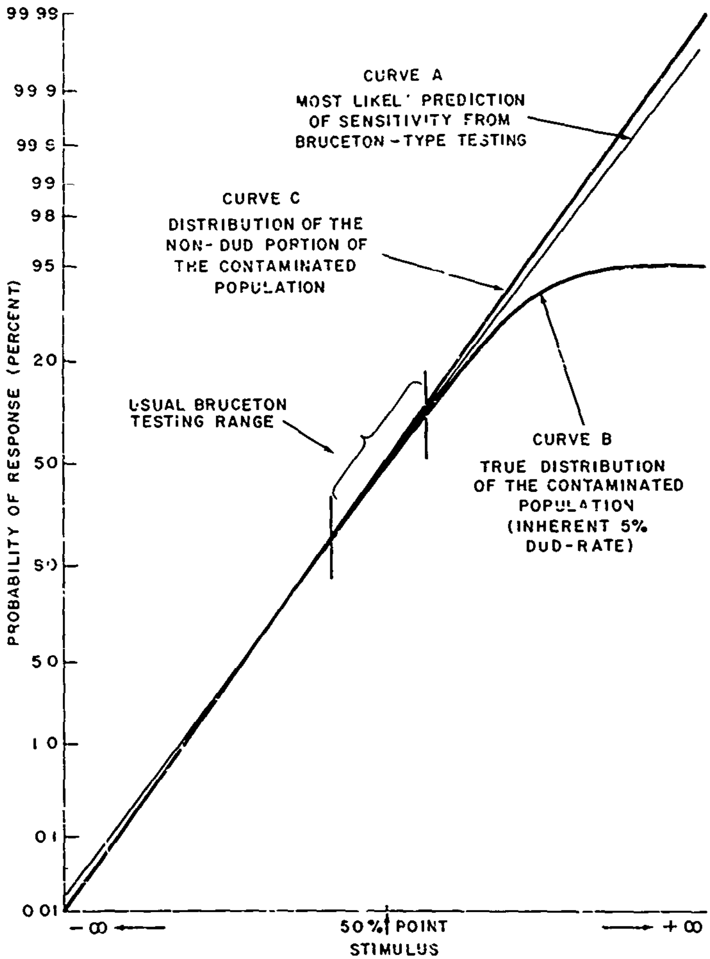

FIGURE 12. FAILURE OF BRUCETON TEST AND ANALYSIS TO DETECT AN INHERENT DUD RATE

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